

Weak Forms of Bioperation-Preopen Sets

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Abstract: In this paper we introduce and study the notions minimal $[\gamma, \gamma']$ -preclosed sets and maximal $[\gamma, \gamma']$ -preopen sets in a topological space (X, τ) .

Keywords: minimal $[\gamma, \gamma']$ -preclosed sets and maximal $[\gamma, \gamma']$ -preopen sets.

1. INTRODUCTION

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc.by utilizing generalized open sets. Kasahara [1] defined the concept of an operation on topological spaces. Maki and Noiri [3] introduced the notions of $\tau_{[\gamma, \gamma']}$, which is the collection of all $[\gamma, \gamma']$ -open sets in a topological space. In this paper we introduce and study the notions (X, τ) minimal $[\gamma, \gamma']$ -preclosed sets and maximal $[\gamma, \gamma']$ -preopen sets in a topological space (X, τ) .

2. PRELIMINARIES

The closure and interior of a subset A of (X, τ) are denoted by $Cl(A)$ and $Int(A)$, respectively.

Definition 2.1

[1] Let (X, τ) be a topological space. An operation γ on the topology τ is function from τ on to power set $P(X)$ of X such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of τ at V . It is denoted by $\gamma: \tau \rightarrow P(X)$.

Definition 2.2

[3] A topological space (X, τ) equipped with two operations namely γ and γ' defined on τ is called a bioperation-topological space and it is denoted by $(X, \tau, \gamma, \gamma')$.

Definition 2.3

A subset A of a topological space (X, τ) is said to be $[\gamma, \gamma']$ -open set is [3] if for each $x \in A$ there exist open neighbourhoods U and V of x such that $U^\gamma \cap V^{\gamma'} \subset A$. The complement of a $[\gamma, \gamma']$ -open set is called a $[\gamma, \gamma']$ -closed set. Also $\tau_{[\gamma, \gamma']}$ denotes set of all $[\gamma, \gamma']$ -open sets in (X, τ) .

Definition 2.4

[3] For a subset A of (X, τ) , $\tau_{[\gamma, \gamma']}$ -Cl(A) denotes the intersection of all $[\gamma, \gamma']$ -closed sets containing A, that is $\tau_{[\gamma, \gamma']}$ -Cl(A) = $\bigcap \{F : A \subset F, X \setminus F \in \tau_{[\gamma, \gamma']}\}$.

Definition 2.5

Let A be any subset of X. The $\tau_{[\gamma, \gamma']}$ -Int(A) is defined as $\tau_{[\gamma, \gamma']}$ -Int(A) = $\bigcup \{U : U \text{ is a } [\gamma, \gamma']\text{-open set and } U \subset A\}$

Definition 2.6

A subset A of a topological space (X, τ) is said to be $[\gamma, \gamma']$ -preopen [2] if $A \subset \tau_{[\gamma, \gamma']}$ -Int($\tau_{[\gamma, \gamma']}$ -Cl(A)).

3. MAXIMAL AND MINIMAL SETS VIA BIOPERATION-SEMIOPEN SETS

In this section, we introduce and study minimal $[\gamma, \gamma']$ -preclosed sets and maximal $[\gamma, \gamma']$ -preopen sets in a bioperation-topological space $(X, \tau, \gamma, \gamma')$.

Definition 3.1

A proper nonempty $[\gamma, \gamma']$ -preclosed subset F of a bioperation-topological space $(X, \tau, \gamma, \gamma')$ is said to be a minimal $[\gamma, \gamma']$ -preclosed set if any $[\gamma, \gamma']$ -preclosed set contained in F is ϕ or F.

Definition 3.2

A proper nonempty $[\gamma, \gamma']$ -preopen U of a bioperation-topological space $(X, \tau, \gamma, \gamma')$

is said to be a maximal $[\gamma, \gamma']$ -preopen set if any $[\gamma, \gamma']$ -preopen set containing U is either X or U.

The following theorem shows the relation between minimal $[\gamma, \gamma']$ -preclosed sets and maximal $[\gamma, \gamma']$ -preopen sets

Theorem 3.3. A proper nonempty subset U of a bioperation-topological space $(X, \tau, \gamma, \gamma')$ is a maximal $[\gamma, \gamma']$ -preopen if and only if $X \setminus U$ is minimal $[\gamma, \gamma']$ -preclosed.

Proof. Let U be a maximal $[\gamma, \gamma']$ -preopen set. Suppose $X \setminus U$ is not minimal $[\gamma, \gamma']$ -preclosed set. Then there exists a $[\gamma, \gamma']$ -preclosed set $V \neq X \setminus U$ such that $\phi \neq V \subset X \setminus U$. That is $U \subset X \setminus U$ and $X \setminus V$ is a $[\gamma, \gamma']$ -preopen set, which is a contradiction for U is a minimal $[\gamma, \gamma']$ -preclosed set. Conversely, let $X \setminus U$ be a minimal $[\gamma, \gamma']$ -preclosed set. Suppose U is not a maximal $[\gamma, \gamma']$ -preopen set. Then there exists a $[\gamma, \gamma']$ -preopen set $E \neq U$ such that $U \subset E \neq X$. That is $\phi \neq X \setminus E \subset X \setminus U$ and $X \setminus E$ is a $[\gamma, \gamma']$ -preclosed set, which is a contradiction for $X \setminus U$ is a minimal $[\gamma, \gamma']$ -preclosed set. Therefore, U is a maximal $[\gamma, \gamma']$ -preclosed set.

Lemma 3.4. For the subsets U and V be any two subsets of a bioperation-topological space $(X, \tau, \gamma, \gamma')$, we have the following:

- (1) If U is minimal $[\gamma, \gamma']$ -preclosed and V a $[\gamma, \gamma']$ -preclosed set, then $U \cap V = \phi$ or $U \subset V$.
- (2) If U and V are minimal $[\gamma, \gamma']$ -preclosed sets, then $U \cap V = \phi$ or $U = V$.

Proof. (1). If $U \cap V \neq \phi$, then there is nothing to prove. If $U \cap V = \phi$, then $U \cap V \subset U$. Since U is a minimal $[\gamma, \gamma']$ -preclosed set, $U \cap V = U$. Hence $U \subset V$.
 (2). If $U \cap V \neq \phi$, then $U \subset V$ and $V \subset U$ by (1). Hence $U = V$.

Theorem 3.5. Let U be a minimal $[\gamma, \gamma']$ -preclosed subset of a bioperation-topological space $(X, \tau, \gamma, \gamma')$. If $x \in U$, then $U \subset W$ for some $[\gamma, \gamma']$ -preclosed set W containing x.

Proof. Let $x \in U$ and W be a $[\gamma, \gamma']$ -preclosed set such that $x \in W$. Then $U \cap W \neq \phi$. By Lemma 3.4(1), $U \subset W$.

Theorem 3.6. Let U be a minimal $[\gamma, \gamma']$ -preclosed subset of a bioperation-topological space $(X, \tau, \gamma, \gamma')$. Then $U = \bigcap \{W : W \in [\gamma, \gamma']\text{-}SC(X, x)\}$.

Proof. By Theorem 3.5 and U is a $[\gamma, \gamma']$ -preclosed set containing x, we have $U \subset \bigcap \{W : W \in [\gamma, \gamma']\text{-}SC(X, x)\}$. Next let, $x \in \bigcap \{W : W \in [\gamma, \gamma']\text{-}SC(X, x)\}$. This implies that, $x \in W$ for all $[\gamma, \gamma']$ -preclosed set W. As U is $[\gamma, \gamma']$ -preclosed, $x \in U$; hence $\bigcap \{W : W \in [\gamma, \gamma']\text{-}SC(X, x)\} = U$.

Theorem 3.7. Let U be a nonempty $[\gamma, \gamma']$ -preclosed subset of a bioperation-topological space $(X, \tau, \gamma, \gamma')$. Then the following statements are equivalent:

- (1) U is a minimal $[\gamma, \gamma']$ -preclosed set.
- (2) $U \subset [\gamma, \gamma']\text{-}pCl(S)$ for any nonempty subset S of U.
- (3) $[\gamma, \gamma']\text{-}pCl(U) = [\gamma, \gamma']\text{-}pCl(S)$ for any nonempty subset S of U.

Proof. (1) \Rightarrow (2): Let $x \in U$; U be a minimal $[\gamma, \gamma']$ -preclosed set and $S (\neq \phi) \subset U$. By Theorem 3.5, for any $[\gamma, \gamma']$ -preclosed set W containing x, $S \subset U \subset W$ gives $S \subset W$. Now $S = S \cap U \subset S \cap W$. Since $S \neq \phi$, $S \cap W \neq \phi$. Since W is any $[\gamma, \gamma']$ -preclosed set containing x, by Theorem 3.5, $x \in [\gamma, \gamma']\text{-}pCl(S)$. That is, $x \in U \Rightarrow x \in [\gamma, \gamma']\text{-}pCl(S)$. Hence $U \subset [\gamma, \gamma']\text{-}pCl(S)$ for any nonempty subset S of U.

(2) \Rightarrow (3): Let S be a nonempty subset of U. Then $[\gamma, \gamma']\text{-}pCl(S) \subset [\gamma, \gamma']\text{-}pCl(U)$. By (2), $[\gamma, \gamma']\text{-}pCl(U) \subset [\gamma, \gamma']\text{-}pCl([\gamma, \gamma']\text{-}pCl(S)) = [\gamma, \gamma']\text{-}pCl(S)$.

That is, $[\gamma, \gamma']\text{-}pCl(U) \subset [\gamma, \gamma']\text{-}pCl(S)$. We have $[\gamma, \gamma']\text{-}pCl(U) = [\gamma, \gamma']\text{-}pCl(S)$ for any nonempty subset S of U.

(3) \Rightarrow (1): Suppose U is not a minimal $[\gamma, \gamma']$ -preclosed set. Then there exists a nonempty $[\gamma, \gamma']$ -preclosed set V such that $V \subset U$ and $V \neq U$. Now, there exists an element a in U such that $a \notin V$. That

is ,
 $[\gamma, \gamma']-pCl(\{a\}) \subset [\gamma, \gamma']-pCl(X \setminus V) = X \setminus V$,
 as $X \setminus V$ is $[\gamma, \gamma']$ -preclosed in X . It follows that
 $[\gamma, \gamma']-pCl(\{a\}) \neq [\gamma, \gamma']-pCl(U)$. This is a
 contradiction for
 $[\gamma, \gamma']-pCl(\{a\}) = [\gamma, \gamma']-pCl(U)$ for any
 $\{a\} (\neq \phi) \subset U$. Therefore, U is a minimal $[\gamma, \gamma']$ -
 preclosed set.

Theorem 3.8. *If V is a nonempty finite $[\gamma, \gamma']$ -
 preclosed subset of a bioperation-topological
 space $(X, \tau, \gamma, \gamma')$, then there exists at least one
 (finite) minimal $[\gamma, \gamma']$ -preclosed set U such that
 $U \subset V$.*

Proof. If V is a minimal $[\gamma, \gamma']$ -preclosed set, we
 may set $U=V$. If V is not a minimal $[\gamma, \gamma']$ -preclosed
 set, then there exists a (finite) $[\gamma, \gamma']$ -preclosed set
 V_1 such that $\phi = V_1 \subset V$. If V_1 is a minimal $[\gamma, \gamma']$ -
 preclosed set, we may $U=V_1$. If V_1 is not a minimal
 $[\gamma, \gamma']$ -preclosed set, then there exists a (finite)
 $[\gamma, \gamma']$ -preclosed set V_2 such that $\phi = V_2 \subset V_1$.
 Continuing this process, we have a sequence of
 $[\gamma, \gamma']$ -preclosed sets
 $V \supset V_1 \supset V_2 \supset V_3 \supset \dots \supset V_k \supset \dots$. Since V is
 a finite set, this process repeats only finitely many
 times and finally we get a minimal $[\gamma, \gamma']$ -preclosed
 set $U=V_n$ for some positive integer n .

Theorem 3.9. *Let U and U_α be minimal $[\gamma, \gamma']$ -
 preclosed subsets of a bioperation-topological space
 $(X, \tau, \gamma, \gamma')$ for any element $\alpha \in \Delta$. If $U \subset \bigcup_{\alpha \in \Delta} U_\alpha$,
 then there exists an element $\alpha \in \Delta$ such that
 $U = U_\alpha$.*

Proof. Let $U \subset \bigcup_{\alpha \in \Delta} U_\alpha$. Then $U \cap (\bigcup_{\alpha \in \Delta} U_\alpha) = U$.
 That is $\bigcup_{\alpha \in \Delta} (U \cap U_\alpha) = U$. Also by lemma 3.4 (2),
 $U \cap U_\alpha = \phi$ or $U = U_\alpha$ for any $\alpha \in \Delta$. It follows
 that there exists an element $\alpha \in \Delta$ such that
 $U = U_\alpha$.

Theorem 3.10. *Let U and U_α be minimal $[\gamma, \gamma']$ -
 preclosed subsets of a bioperation-topological space
 $(X, \tau, \gamma, \gamma')$ for any $\alpha \in \Delta$. If $U \neq U_\alpha$ for any
 $\alpha \in \Delta$, then $(\bigcup_{\alpha \in \Delta} U_\alpha) \cap U = \phi$.*

Proof. Suppose that $(\bigcup_{\alpha \in \Delta} U_\alpha) \cap U = \phi$. Then
 there exists an element $\alpha \in \Delta$ such that
 $U \cap U_\alpha = \phi$. By Lemma 3.4 (2), we have
 $U = U_\alpha$, Which contradicts the fact $U \neq U_\alpha$
 for any $\alpha \in \Delta$. Hence $(\bigcup_{\alpha \in \Delta} U_\alpha) \cap U = \phi$.

Theorem 3.11. *Let A and B be any two subsets of
 a bioperation-topological space $(X, \tau, \gamma, \gamma')$
 .Then we have the following:*

- (1) *If A is maximal $[\gamma, \gamma']$ -preopen set and
 B a $[\gamma, \gamma']$ -preopen set, then
 $A \cup B = X$ or $B \subset A$.*
- (2) *Let A and B are maximal $[\gamma, \gamma']$ -
 preopen set, then $A \cup B = X$ or $A = B$.*

Proof. (1). If $A \cup B = X$, then there is nothing
 to prove. If $A \cup B \neq X$, then $A \cup B$ is a $[\gamma, \gamma']$ -
 preopen set such that $A \subset A \cup B$. Then
 $A \cup B = A$. Hence $B \subset A$.

(2). If $A \cup B \neq X$, then $A \cup B$ is a $[\gamma, \gamma']$ -
 preopen set such that $A, B \subset A \cup B$, that is,
 $A \cup B = A$ and $A \cup B = B$. Hence $A = B$.

Theorem 3.12. *Let F be a maximal $[\gamma, \gamma']$ -
 preopen subsets of a bioperation-topological space
 $(X, \tau, \gamma, \gamma')$. If $x \in F$, then $S \subset F$ for some
 $[\gamma, \gamma']$ -preopen set S containing x .*

Proof. Similar to the proof of Theorem 3.5.

Theorem 3.13. *Let A, B and C be three $[\gamma, \gamma']$ -
 preopen sets of a bioperation-topological space
 $(X, \tau, \gamma, \gamma')$ such that $A \neq B$. If $A \cap B \subset C$,
 then either $A = C$ or $B = C$.*

Proof. If $A = C$, then there is nothing to prove.
 If $A \neq C$, then we have to prove $B=C$.

Now $B \cap C = B \cap (C \cap X)$
 $= B \cap (C \cap (A \cup B))$ (by Theorem 3.11 (2))
 $= B \cap ((C \cap A) \cup (C \cap B))$
 $= (B \cap C \cap A) \cup (B \cap C)$
 $= (A \cap B) \cup (C \cap B)$
 $= (A \cap C) \cap B$

$= X \cap B = B$ (Since A and C are maximal $[\gamma, \gamma']$ -preopen set by Theorem 3.11 (2), $A \cup C = X$).

That is, $B \cap C = B \Rightarrow B \subset C$. Since B and C are maximal $[\gamma, \gamma']$ -preopen sets, we have $B = C$. Hence $B = C$.

Theorem 3.14. Let $(X, \tau, \gamma, \gamma')$ be a bioperation-topological space. If A, B and C are maximal $[\gamma, \gamma']$ -preopen sets which are different from each other, then $(A \cap B) \not\subseteq (A \cap C)$.

Proof. Let $A \cap B \subset A \cap C$. Then $(A \cap B) \cup (C \cap B) \subset (A \cap C) \cup (C \cap B)$. That is, $(A \cap C) \cap B \subset C \cap (A \cup B)$. By Theorem 3.11 (2), $A \cup C = X = A \cup B$. Hence $X \cap B \subset C \cap X \Rightarrow B \subset C$. Thus from the definition of maximal $[\gamma, \gamma']$ -preopen sets, we have $B = C$, which is a contradiction to the fact that A, B and C are different to each other. Therefore, $(A \cap B) \not\subseteq (A \cap C)$.

Theorem 3.15. Let $(X, \tau, \gamma, \gamma')$ be a bioperation-topological space. If F is a maximal $[\gamma, \gamma']$ -preopen set and x be an element of F, then $F = \bigcup \{S : S \text{ is an } [\gamma, \gamma']\text{-preopen set } E \text{ such that } F \cup S \neq X\}$.

Proof. Similar to the proof of Theorem 3.6.

Theorem 3.16. Let $(X, \tau, \gamma, \gamma')$ be a bioperation-topological space. If F is proper nonempty cofinite $[\gamma, \gamma']$ -preopen set, then there exists (cofinite) maximal $[\gamma, \gamma']$ -preopen set E such that $F \subset E$.

Proof. If F is a maximal $[\gamma, \gamma']$ -preopen set, we may set $E = F$. If F is not a maximal $[\gamma, \gamma']$ -preopen set, then there exists a (cofinite) $[\gamma, \gamma']$ -preopen set F_1 such that $F \subset F_1 \neq X$. If F_1 is a maximal $[\gamma, \gamma']$ -preopen set, we may set $E = F_1$. If F_1 is not a maximal $[\gamma, \gamma']$ -preopen set, then there exists a (cofinite) $[\gamma, \gamma']$ -preopen set $F_2 \neq X$ such that $F \subset F_1 \subset F_2 (\neq X)$. Continuing this process, we have a sequence of $[\gamma, \gamma']$ -preopen sets such that $F \subset F_1 \subset F_2 \subset \dots \subset F_k \subset \dots$. Since F is a cofinite set, this process repeats only finitely many times and finally we get a maximal $[\gamma, \gamma']$ -preopen set $E = F$.

Theorem 3.17. Let $(X, \tau, \gamma, \gamma')$ be a bioperation-topological space. Then we have the following:

- (1) Let A be a maximal $[\gamma, \gamma']$ -preopen set and $x \in X \setminus A$. Then $X \setminus A \subset B$ for any $[\gamma, \gamma']$ -preopen set B containing x.
- (2) Let A be a maximal $[\gamma, \gamma']$ -preopen set. Then either of the following (i) or (ii) holds:
 - (i) For each $x \in X \setminus A$ and each $[\gamma, \gamma']$ -preopen set B containing x, $B = X$.
 - (ii) There exists a $[\gamma, \gamma']$ -preopen set B such that $X \setminus A \subset B$.
- (3) Let A be a maximal $[\gamma, \gamma']$ -preopen set. Then either of the following (i) or (ii) holds:
 - (i) For each $x \in X \setminus A$ and each $[\gamma, \gamma']$ -preopen set B containing x, $X \setminus A \subset B$.
 - (ii) There exists a $[\gamma, \gamma']$ -preopen set B such that $X \setminus A = B$.

Proof. (1). Since $x \in X \setminus A$, we have $B \not\subseteq A$ for any $[\gamma, \gamma']$ -preopen set B containing x. Then by Theorem 3.11 (1), $A \cup B = X \Rightarrow X \setminus A \subset B$.

(2). If (i) holds, we are done. Let (i) do not hold. Then there exists an element $x \in X \setminus A$ and a $[\gamma, \gamma']$ -preopen set B containing x such that $B \subset X$. Then by Theorem 3.11 (1), $A \cup B = X$ or $B \subset A$. But $B \not\subseteq A \Rightarrow A \cup B = X \Rightarrow X \setminus A \subset B$.

(3). If (ii) holds, we are done. Let (ii) do not hold. Then (by (i)) for each $x \in X \setminus A$ and each $[\gamma, \gamma']$ -preopen set B containing x, $X \setminus A \subset B$. Hence by assumption $X \setminus A \subset B$.

Theorem 3.18. Let A be a maximal $[\gamma, \gamma']$ -preopen sets in a bioperation-topological space $(X, \tau, \gamma, \gamma')$. Then either $[\gamma, \gamma']\text{-}pCl(A) = X$ or $[\gamma, \gamma']\text{-}pCl(A) = A$.

Proof. Since A is maximal $[\gamma, \gamma']$ -preopen set, only the following cases (i) and (ii) occur by Theorem 3.17 (3).

- (i) For each $x \in X$ and $x \in X \setminus A$ and each $[\gamma, \gamma']$ -preopen set B containing x , we have, $X \setminus A \subset B$: Let $x \in X \setminus A$ and B be any $[\gamma, \gamma']$ -preopen set B containing x . Since $X \setminus A \neq B$, we have $B \cap A \neq \emptyset$ and hence $X \setminus A \subset [\gamma, \gamma']-pCl(A)$. Since $X = A \cup (X \setminus A)$
 $\subset A \cup [\gamma, \gamma']-pCl(A)$
 $= [\gamma, \gamma']-pCl(A) \subset X$,
 $X = [\gamma, \gamma']-pCl(A)$.

There exists a $[\gamma, \gamma']$ -preopen set B such that $X \setminus A = B (\neq X)$: Since $X \setminus A = B$, a $[\gamma, \gamma']$ -preopen set, A is a $[\gamma, \gamma']$ -preclosed set $\Rightarrow [\gamma, \gamma']-pCl(A)$.

Theorem 3.19. Let A be a maximal $[\gamma, \gamma']$ -preopen sets in a bioperation-topological space $(X, \tau, \gamma, \gamma')$. Then either $[\gamma, \gamma']-pInt(X \setminus A) = X \setminus A$ or $[\gamma, \gamma']-pInt(X \setminus A) = \emptyset$.

Proof. By Theorem 3.18, we have $[\gamma, \gamma']-pCl(A) = A$ or $[\gamma, \gamma']-pCl(A) = X$. That is $[\gamma, \gamma']-pInt(X \setminus A) = X \setminus A$ or $[\gamma, \gamma']-pInt(X \setminus A) = \emptyset$.

Theorem 3.20. Let A be a maximal $[\gamma, \gamma']$ -preopen sets and B be a nonempty subset of $X \setminus A$ in a bioperation-topological space $(X, \tau, \gamma, \gamma')$. Then $[\gamma, \gamma']-pCl(B) = X \setminus A$.

Proof. Since $\emptyset \neq B \subset X \setminus A, W \cap B \neq \emptyset$ for any element $x \in X \setminus A$ and any $[\gamma, \gamma']$ -preopen set W containing x , by Theorem 3.17 (1). Thus, $X \setminus A \subset [\gamma, \gamma']-pCl(B)$. Since $X \setminus A$ is $[\gamma, \gamma']$ -preclosed set and $B \subset X \setminus A$, we have $[\gamma, \gamma']-pCl(B) \subset X \setminus A$.

Corollary 3.21. Let A be a maximal $[\gamma, \gamma']$ -preopen sets in a bioperation-topological space $(X, \tau, \gamma, \gamma')$ and $A \subset B$. Then $[\gamma, \gamma']-pCl(B) = X$.

Proof. Since $A \subset B \subset X$, there exist a nonempty subset F of $X \setminus A$ such that $B = A \cup F$. Hence we have,
 $[\gamma, \gamma']-pCl(B) = [\gamma, \gamma']-pCl(A \cup F)$
 $\supset [\gamma, \gamma']-pCl(A) \cup [\gamma, \gamma']-pCl(F)$
 $\supset A \cup (X \setminus A) = X$ (by Theorem 3.18)

Theorem 3.22. Let A be a maximal $[\gamma, \gamma']$ -preopen sets in a bioperation-topological space $(X, \tau, \gamma, \gamma')$ and let $X \setminus A$ have at least two elements. Then $[\gamma, \gamma']-pCl(X \setminus \{a\}) = X$ for any element a of $X \setminus A$.

Proof. As $A \subset X \setminus \{a\}$, we have, by Corollary 3.21, $[\gamma, \gamma']-pCl(X \setminus \{a\}) = X$.

Theorem 3.23. Let A be a maximal $[\gamma, \gamma']$ -preopen set and G be a proper subset of a bioperation-topological space $(X, \tau, \gamma, \gamma')$ with $A \subset G$. Then $[\gamma, \gamma']-pInt(G) = A$.

Proof. If $G = A$, then $[\gamma, \gamma']-pInt(G) = [\gamma, \gamma']-pInt(A) = A$. If $G \neq A$, then we have $A \subset G$. Thus $A \subset [\gamma, \gamma']-pInt(G)$. Since A is maximal $[\gamma, \gamma']$ -preopen, we also have $[\gamma, \gamma']-pInt(G) \subset A$. Hence $[\gamma, \gamma']-pInt(G) = A$.

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